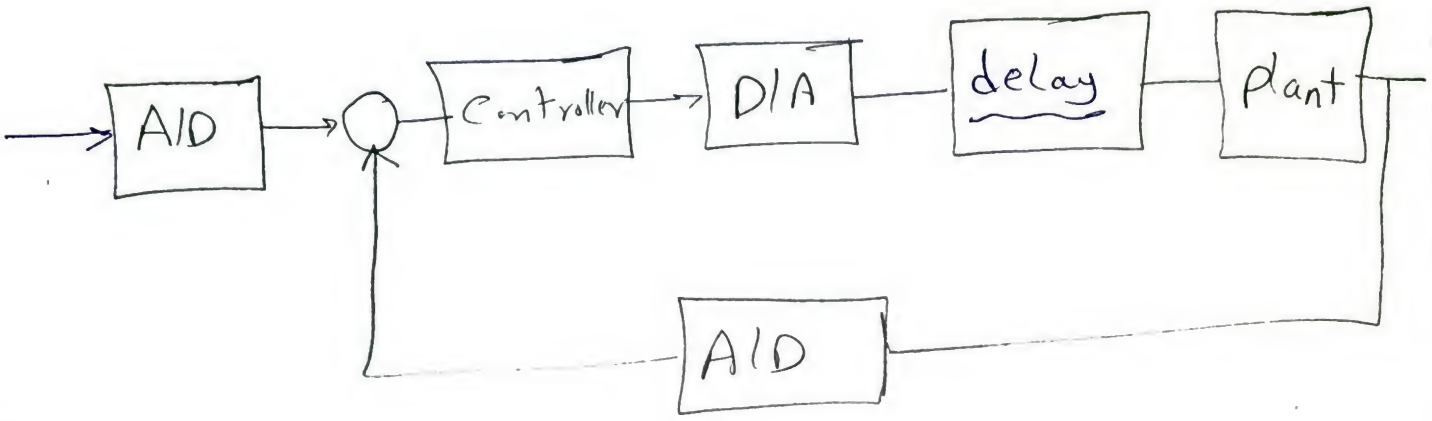
Digital Control sec 9



→ C.T. Plant
(continuous time plant)

C.T. Plant

Discretization → DT-Plant → Design → Digital Controller

design → Controller → discretization → Digital Controller

A/D , D/A

For every control system

1) stability

2) Tracking (steady state error)

ال Sys. بيعمل ال (tracking) (desired o/p) ولا لأ.

3) Robustuess (Disturbance Rejection)

→ steady state error

for unit step $e_{ss} = \frac{1}{1+K_p}$ for type 0

for unit ramp $e_{ss} = \frac{1}{K_v}$ for type 1

for parabolic $e_{ss} = \frac{1}{K_a}$ for type 2

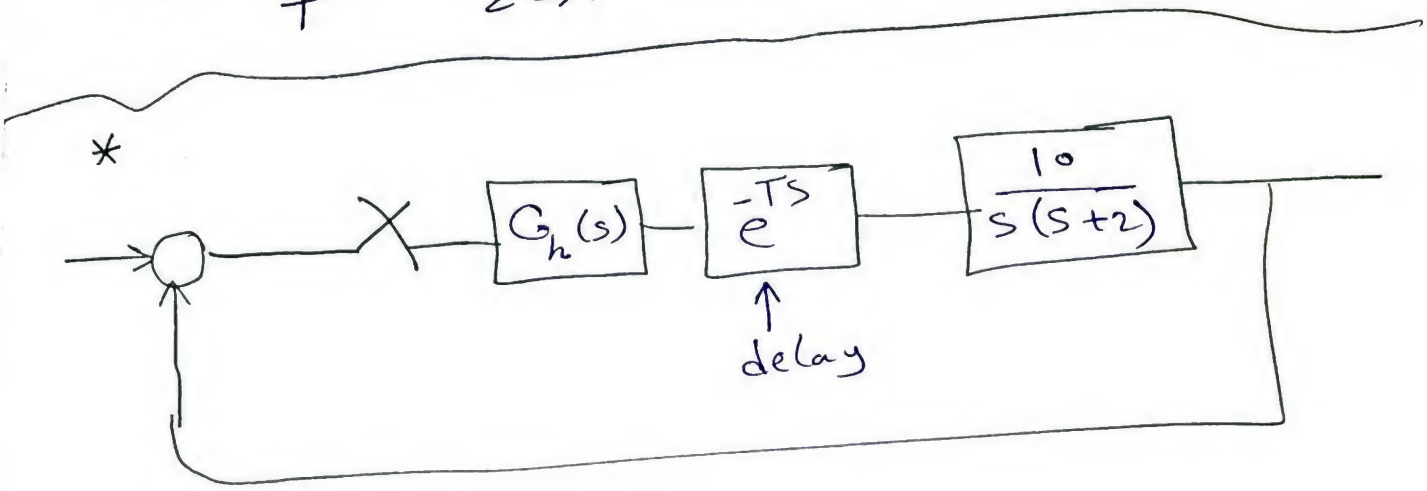
← ال (Final value) في (s-domain) مختلفة عن

اللي في (z-domain)

$$K_p = \frac{1}{T} \lim_{z \to 1} \overline{GH}(z)$$

$$K_v = \frac{1}{T} \lim_{z \to 1} (1 - z^{-1}) \overline{GH}(z)$$

$$K_a = \frac{1}{T^2} \lim_{z \to 1} (1 - z^{-1})^2 \, \overline{GH(z)}$$



$$GH(s) = \left(\frac{1 - e^{-TS}}{s}\right) e^{-TS} * \frac{10}{(s+2)s}$$

$$GH(z) = z^{-1}(1 - z^{-1}) \, Z\left[\frac{10}{s^2(s+2)}\right]$$

unit delay $\rightsquigarrow e^{TS}$; unit delay in z-Transform = z

$$Z\left[\frac{A_1}{s^2} + \frac{\beta}{s} + \frac{c}{s+2}\right]$$

$$Z\left[\frac{5}{s^2} - \frac{2.5}{s} + \frac{2.5}{s+2}\right]$$

$$Z\left[5t - 2.5 + 2.5\, e^{-2t}\right]$$

$$\mathcal{Z}\left[5KT - 2.5 + 2.5\, e^{-2KT}\right]$$

$$= \frac{5ZT}{Z-1} - 2.5\,\frac{Z}{Z-1} + 2.5\,\frac{Z}{Z-e^{-2T}}$$

~~GH(z) =~~

~~GH(z) =~~

$$GH(z) = \frac{z^{-1}(1-z^{-1})\left[5ZT\left(Z-e^{-2T}\right) - 2.5\,(Z-1)\left(Z-e^{-2T}\right) + 2.5\,(Z-1)^2\right]}{(Z-1)^2\left(Z-e^{-2T}\right)}$$

(the term $-2.5(Z-1)(Z-e^{-2T})$ is annotated "$\times Z$")

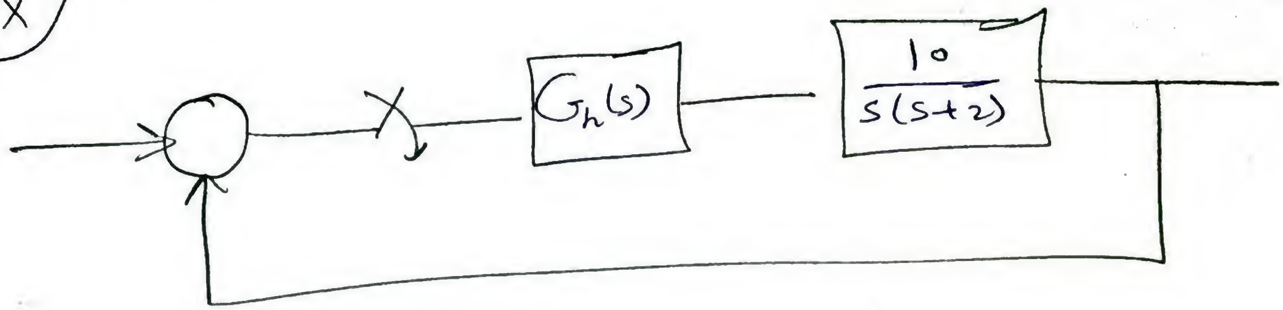
$$GH(z)\Big|_{C.L} = \frac{G(z)}{1+GH(z)}$$

ــ (Physically) ال sys. ده (type 2)

لكن فى ال ،، مفيش حاجة اسمها أشيل (root) فى

البسط مع المقام لكن رياضياً عادى.

ex



stability
↳ c/c eqn.

Stability
→ routh (in Cont.)
→ Jury test
→ check system Poles.

بس حتى في ال (discrete) بنحل Cont. برده لكننا
بنحولها z-domain لـ r-domain

Routh

$$Z = e^{Ts} = \frac{e^{\frac{T}{2}s}}{e^{(-T/2)s}} \simeq \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s} \simeq \frac{1+r}{1-r}$$

map any $z \longrightarrow \frac{1+r}{1-r}$

Jury test

1) $F(1) > 0$ — cle eqr $|_{z=1}$

2) $(-1)^n F(-1) > 0$ — n : order

3) $a_0 < a_n$

→ Jury matrix

a_0	a_1	- - - - -		a_n
a_n	a_{n-1}	- - -		a_0
$\begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix}$	$\begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$	. . .		

⋮

(ex)

$P(z) = z^2 + 1.703z + 1.620$ c/c

i) $P(1) = 1 + 1.703 + 1.620 = .\ 7\ 0$

2) $(-1)^n P(-1) = (-1)^2 [1 - 1.703 + 1.620]\ 7\ 0$

3) $a_0 = 1.620 \qquad a_0 \ 7\ a_2$

$a_2 = 1$

⟶ system unstable

---

c/c eqn:

$z^3 - 1.368z^2 + 0.368z + 0.368Kz + 0.264K = 0$

$F(1)\ 7\ 0 \rightarrow 1 - 1.368 + 0.368 + 0.368K$

$+ 0.264K\ 7\ 0$ $\boxed{n=3}$

$(-1)^n F(-1)\ 7\ 0 \rightarrow (-1) - 1.368 - 0.368 - - -$

$\rightarrow a_0 < a_3$

$\downarrow \qquad \downarrow_1$

$0.264K$

المفروض ال ٣ شروط السابقة كل شرط هيطلع range لـ K هنحسب التقاطع ما بينهم

Jury matrix

$0.264K$	$(1+K)0.368$	-1.368	1
1	-1.368	$(1+K)0.368$	$0.264K$
$(0.264K)^2 - 1$	---	$-0.264 * 1.368K - (1+K)0.368$	

← كده أربع معادلات هنحسب التقاطع ما بينهم.

⇒ (range for stability check)

→ you better use Routh array.

Solve by Routh

$$z^3 - 1.368z^2 + 0.368z + 0.368Kz + 0.264Z = 0$$

$$z = \frac{1+r}{1-r}$$

$$\frac{(1+r)^3}{(1-r)^3} - 1.368\left(\frac{1+r}{1-r}\right)^2 + 0.368\left(\frac{1+r}{1-r}\right) + 0.368K\left(\frac{1+r}{1-r}\right) + 0.264\left(\frac{1+r}{1-r}\right)$$

ـــ المعادله هنخلص حساباتها و نحسب بـ (Routh)

---

* 2nd order eqn. ⟶ Jury

* Check stability ⟶ Routh